Signals and Systems - H. P. Hsu, section 5.47

5.47. The most widely used graphical representation of the frequency response $H(\omega)$ is the *Bode plot* in which the quantities $20\log_{10}|H(\omega)|$ and $\theta_H(\omega)$ are plotted versus ω, with ω plotted on a logarithmic scale. The quantity $20\log_{10}|H(\omega)|$ is referred to as the magnitude expressed in *decibels* (dB), denoted by $|H(\omega)|_{\text{dB}}$. Sketch the Bode plots for the following frequency responses:

(a) $H(\omega) = 1 + \dfrac{j\omega}{10}$

(b) $H(\omega) = \dfrac{1}{1 + j\omega/100}$

(c) $H(\omega) = \dfrac{10^4(1 + j\omega)}{(10 + j\omega)(100 + j\omega)}$

(a)
$$|H(\omega)|_{\text{dB}} = 20\log_{10}|H(\omega)| = 20\log_{10}\left|1 + j\frac{\omega}{10}\right|$$

For $\omega \ll 10$,

$$|H(\omega)|_{\text{dB}} = 20\log_{10}\left|1 + j\frac{\omega}{10}\right| \longrightarrow 20\log_{10} 1 = 0 \qquad \text{as } \omega \longrightarrow 0$$

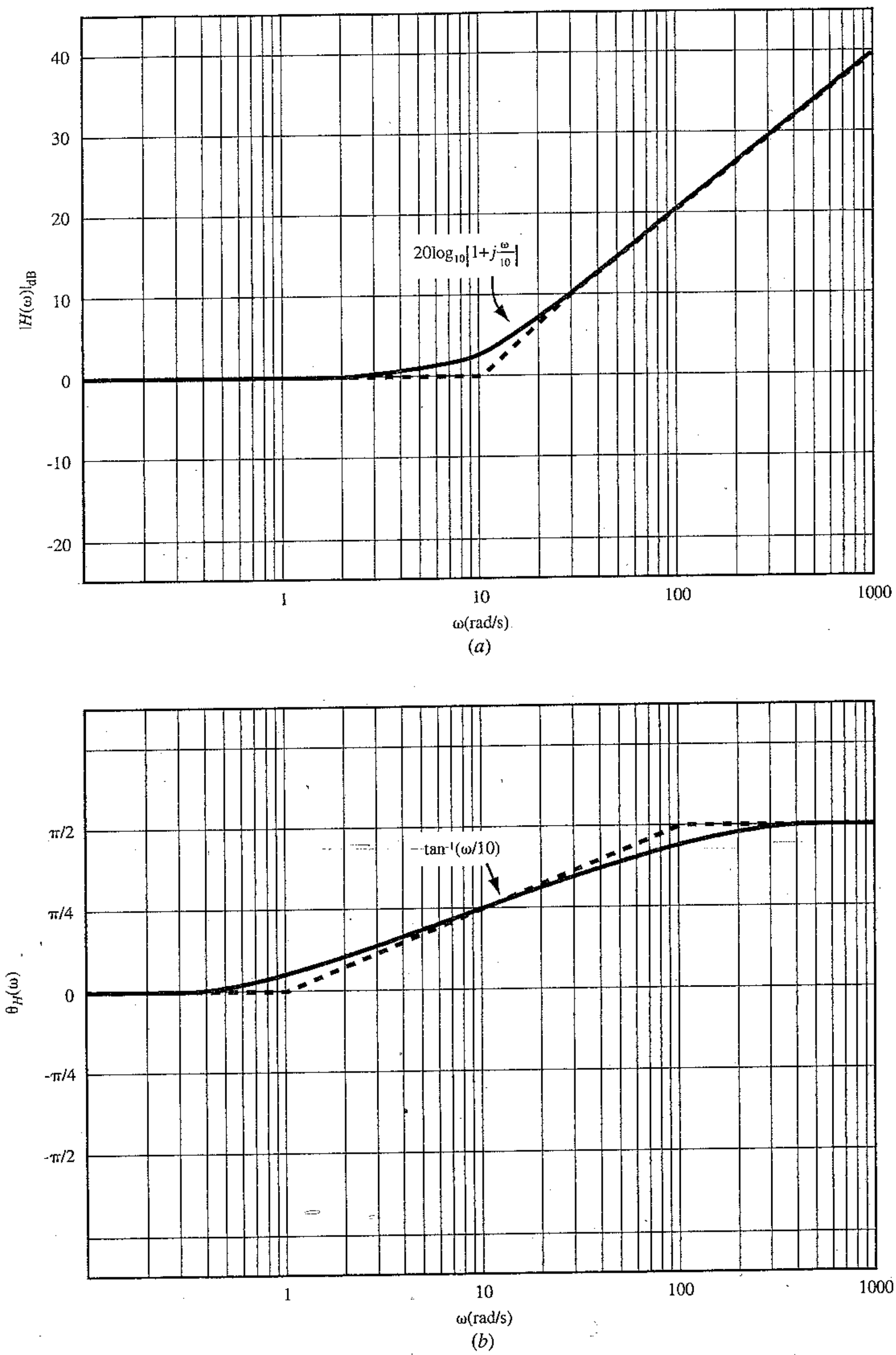


Fig. 5-28 Bode plots.

For $\omega \gg 10$,

$$|H(\omega)|_{\text{dB}} = 20\log_{10}\left|1 + j\frac{\omega}{10}\right| \longrightarrow 20\log_{10}\left(\frac{\omega}{10}\right) \qquad \text{as } \omega \longrightarrow \infty$$

On a log frequency scale, $20\log_{10}(\omega/10)$ is a straight line with a slope of 20 dB/decade (a decade is a 10-to-1 change in frequency). This straight line intersects the 0-dB axis at $\omega = 10$ [Fig. 5-28(*a*)]. (This value of ω is called the *corner frequency*.) At the corner frequency $\omega = 10$

$$H(10)|_{\text{dB}} = 20\log_{10}|1 + j1| = 20\log_{10}\sqrt{2} \approx 3 \text{ dB}$$

The plot of $|H(\omega)|_{\text{dB}}$ is sketched in Fig. 5-28(*a*). Next,

$$\theta_H(\omega) = \tan^{-1}\frac{\omega}{10}$$

Then

$$\theta_H(\omega) = \tan^{-1}\frac{\omega}{10} \longrightarrow 0 \qquad \text{as } \omega \longrightarrow 0$$

$$\theta_H(\omega) = \tan^{-1}\frac{\omega}{10} \longrightarrow \frac{\pi}{2} \qquad \text{as } \omega \longrightarrow \infty$$

At $\omega = 10$, $\theta_H(10) = \tan^{-1}1 = \pi/4$ radian (rad). The plot of $\theta_H(\omega)$ is sketched in Fig. 5-28(*b*). Note that the dotted lines represent the straight-line approximation of the Bode plots.

(*b*)
$$|H(\omega)|_{\text{dB}} = 20\log_{10}\left|\frac{1}{1 + j\omega/100}\right| = -20\log_{10}\left|1 + j\frac{\omega}{100}\right|$$

For $\omega \ll 100$,

$$|H(\omega)|_{\text{dB}} = -20\log_{10}\left|1 + j\frac{\omega}{100}\right| \longrightarrow -20\log_{10}1 = 0 \qquad \text{as } \omega \longrightarrow 0$$

For $\omega \gg 100$,

$$|H(\omega)|_{\text{dB}} = -20\log_{10}\left|1 + j\frac{\omega}{100}\right| \longrightarrow -20\log_{10}\left(\frac{\omega}{100}\right) \qquad \text{as } \omega \longrightarrow \infty$$

On a log frequency scale $-20\log_{10}(\omega/100)$ is a straight line with a slope of -20 dB/decade. This straight line intersects the 0-dB axis at the corner frequency $\omega = 100$ [Fig. 5-29(*a*)]. At the corner frequency $\omega = 100$

$$H(100)|_{\text{dB}} = -20\log_{10}\sqrt{2} \approx -3 \text{ dB}$$

The plot of $|H(\omega)|_{\text{dB}}$ is sketched in Fig. 5-29(*a*). Next

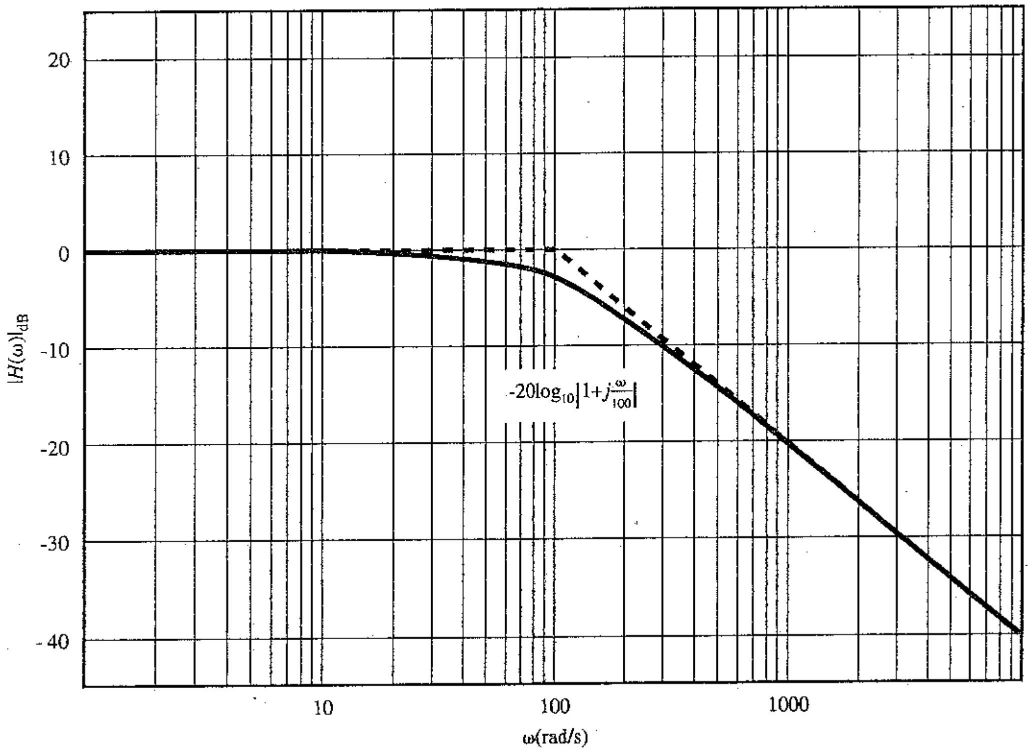
$$\theta_H(\omega) = -\tan^{-1}\frac{\omega}{100}$$

Then

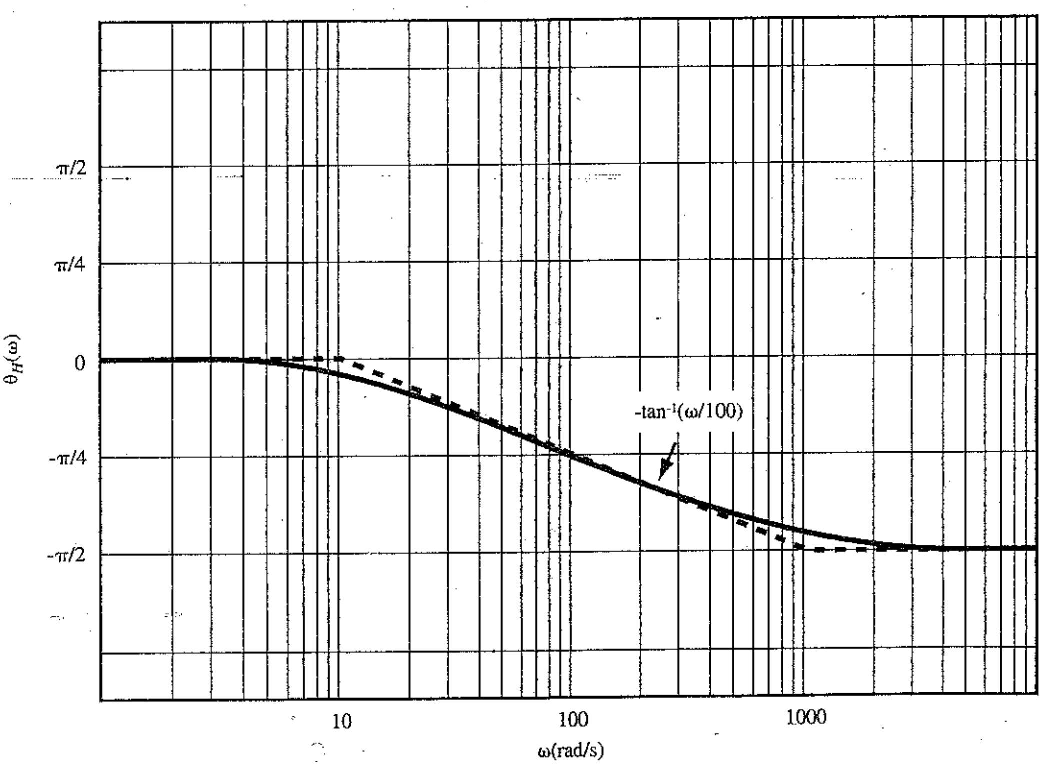
$$\theta_H(\omega) = -\tan^{-1}\frac{\omega}{100} \longrightarrow 0 \qquad \text{as } \omega \longrightarrow 0$$

$$\theta_H(\omega) = -\tan^{-1}\frac{\omega}{100} \longrightarrow -\frac{\pi}{2} \qquad \text{as } \omega \longrightarrow \infty$$

At $\omega = 100$, $\theta_H(100) = -\tan^{-1}1 = -\pi/4$ rad. The plot of $\theta_H(\omega)$ is sketched in Fig. 5-29(*b*).



(a)



(b)

Fig. 5-29 Bode plots.

(c) First, we rewrite $H(\omega)$ in standard form as

$$H(\omega) = \frac{10(1+j\omega)}{(1+j\omega/10)(1+j\omega/100)}$$

Then

$$|H(\omega)|_{\mathrm{dB}} = 20\log_{10}10 + 20\log_{10}|1+j\omega| - 20\log_{10}\left|1+j\frac{\omega}{10}\right| - 20\log_{10}\left|1+j\frac{\omega}{100}\right|$$

Note that there are three corner frequencies, $\omega = 1$, $\omega = 10$, and $\omega = 100$. At corner frequency $\omega = 1$

$$H(1)|_{\mathrm{dB}} = 20 + 20\log_{10}\sqrt{2} - 20\log_{10}\sqrt{1.01} - 20\log_{10}\sqrt{1.0001} \approx 23\ \mathrm{dB}$$

At corner frequency $\omega = 10$

$$H(10)|_{\mathrm{dB}} = 20 + 20\log_{10}\sqrt{101} - 20\log_{10}\sqrt{2} - 20\log_{10}\sqrt{1.01} \approx 37\ \mathrm{dB}$$

At corner frequency $\omega = 100$

$$H(100)|_{\mathrm{dB}} = 20 + 20\log_{10}\sqrt{10{,}001} - 20\log_{10}\sqrt{101} - 20\log_{10}\sqrt{2} \approx 37\ \mathrm{dB}$$

The Bode amplitude plot is sketched in Fig. 5-30(a). Each term contributing to the overall amplitude is also indicated. Next,

$$\theta_H(\omega) = \tan^{-1}\omega - \tan^{-1}\frac{\omega}{10} - \tan^{-1}\frac{\omega}{100}$$

Then

$$\theta_H(\omega) = \longrightarrow 0 - 0 - 0 = 0 \qquad \text{as } \omega \longrightarrow 0$$

$$\theta_H(\omega) = \longrightarrow \frac{\pi}{2} - \frac{\pi}{2} - \frac{\pi}{2} = -\frac{\pi}{2} \qquad \text{as } \omega \longrightarrow \infty$$

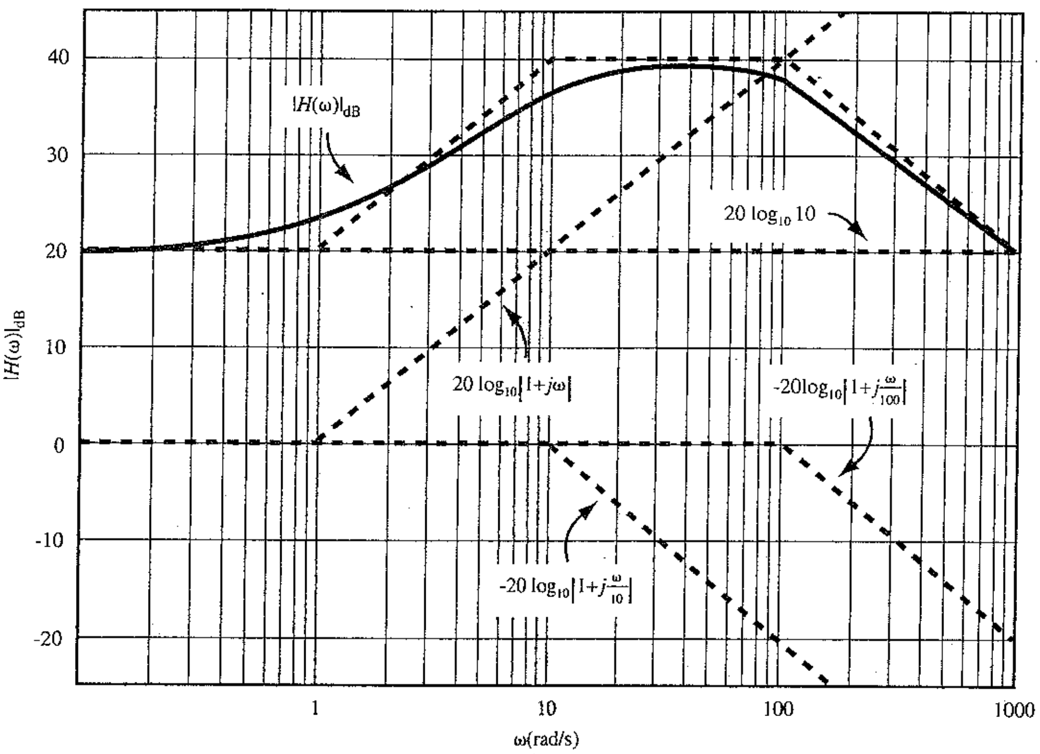
and

$$\theta_H(1) = \tan^{-1}(1) - \tan^{-1}(0.1) - \tan^{-1}(0.01) = 0.676\ \mathrm{rad}$$

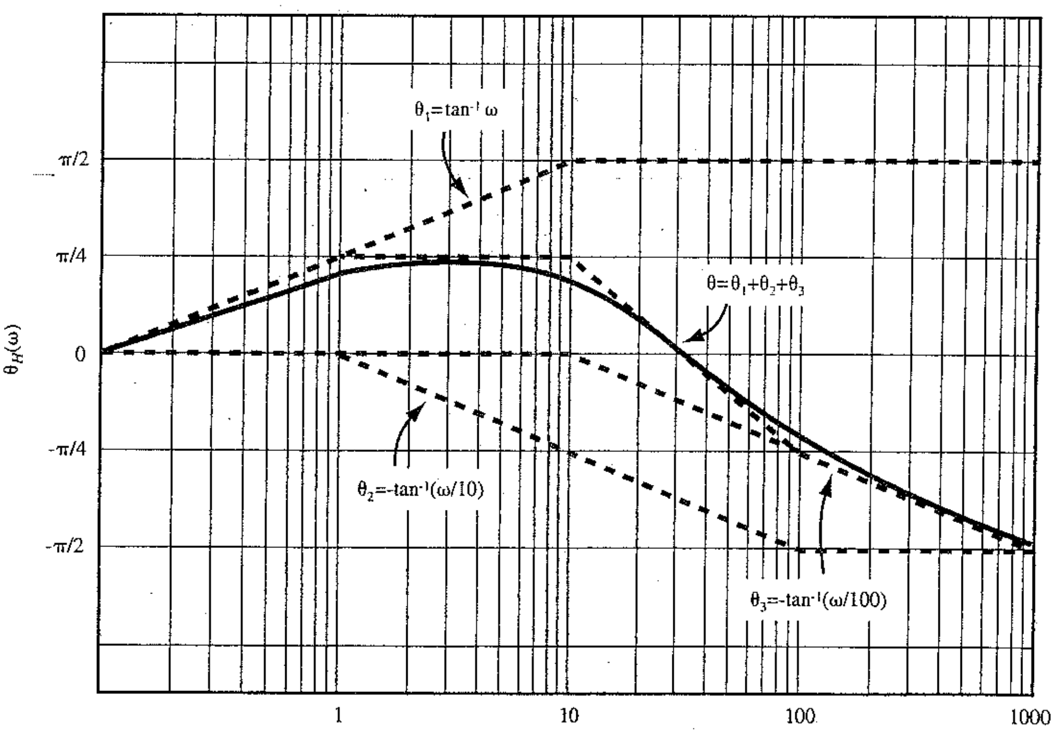
$$\theta_H(10) = \tan^{-1}(10) - \tan^{-1}(1) - \tan^{-1}(0.1) = 0.586\ \mathrm{rad}$$

$$\theta_H(100) = \tan^{-1}(100) - \tan^{-1}(10) - \tan^{-1}(1) = -0.696\ \mathrm{rad}$$

The plot of $\theta_H(\omega)$ is sketched in Fig. 5-30(b).



(a)



(b)

Fig. 5-30 Bode plots.