Useful Formulae:

$$\begin{aligned}
\sin(\theta\pm\phi) &= \sin\theta\cos\phi \pm \cos\theta\sin\phi,\\
\cos\theta\cos\phi &= \frac{1}{2}[\cos(\theta+\phi)+\cos(\theta-\phi)],\\
\sin\theta\sin\phi &= \frac{1}{2}[\cos(\theta-\phi)-\cos(\theta+\phi)],\\
\sin\theta\cos\phi &= \frac{1}{2}[\sin(\theta+\phi)+\sin(\theta-\phi)],\\
\cos^2\theta &= \frac{1}{2}[1+\cos 2\theta],\\
\sin^2\theta &= \frac{1}{2}[1-\cos 2\theta],\\
\cos\theta+\cos\phi &= 2\cos\frac{\theta+\phi}{2}\cos\frac{\theta-\phi}{2},\\
\cos\theta-\cos\phi &= 2\sin\frac{\theta+\phi}{2}\sin\frac{\theta-\phi}{2},\\
\sin\theta\pm\sin\phi &= 2\sin\frac{\theta\pm\phi}{2}\cos\frac{\theta\mp\phi}{2},\\
1 &= \cos^2\theta+\sin^2\theta,\\
1 &= \sec^2\theta-\tan^2\theta,\\
e^{i\theta} &= \cos\theta+i\sin\theta,\\
\cos\theta &= \frac{1}{2}(e^{i\theta}+e^{-i\theta}),\\
\sin\theta &= \frac{1}{2i}(e^{i\theta}-e^{-i\theta}),
\end{aligned}$$

$$\begin{aligned}
f(z) &= \sum_{n=0}^{\infty}\frac{1}{n!}f^{(n)}(a)(z-a)^n,\\
e^z &= 1+z+\frac{1}{2!}z^2+\frac{1}{3!}z^3+\ldots,\\
\cos z &= 1-\frac{1}{2!}z^2+\frac{1}{4!}z^4+\ldots,\\
\sin z &= z-\frac{1}{3!}z^3+\frac{1}{5!}z^5-\ldots,\\
\ln(1+z) &= z-\frac{1}{2}z^2+\frac{1}{3}z^3-\ldots\ [|z|<1],\\
(1+z)^n &= 1+nz+\frac{n(n-1)}{2!}z^2+\ldots\ [|z|<1],\\
\boldsymbol{\nabla} f &= \hat{\mathbf{x}}\frac{\partial f}{\partial x}+\hat{\mathbf{y}}\frac{\partial f}{\partial y}+\hat{\mathbf{z}}\frac{\partial f}{\partial z},\\
\boldsymbol{\nabla}\times\mathbf{A} &= \hat{\mathbf{x}}\left(\frac{\partial A_z}{\partial y}-\frac{\partial A_y}{\partial z}\right)+\hat{\mathbf{y}}\left(\frac{\partial A_x}{\partial z}-\frac{\partial A_z}{\partial x}\right)\\
&\quad+\hat{\mathbf{z}}\left(\frac{\partial A_y}{\partial x}-\frac{\partial A_x}{\partial y}\right),
\end{aligned}$$

$$\begin{aligned}
\int\frac{dx}{1+x^2} &= \arctan x + c,\\
\int\frac{dx}{1-x^2} &= \operatorname{arctanh} x + c,\\
\int\frac{dx}{\sqrt{1-x^2}} &= \arcsin x + c,\\
\int\frac{dx}{\sqrt{1+x^2}} &= \operatorname{arcsinh} x + c,\\
\int\tan x\,dx &= -\ln\cos x + c,\\
\int\tanh x\,dx &= \ln\cosh x + c,\\
\int\frac{dx}{x+x^2} &= \ln\left(\frac{x}{1+x}\right)+c,\\
\int\frac{x\,dx}{x+x^2} &= \frac{1}{2}\ln\left(1+x^2\right)+c,
\end{aligned}$$

$$\begin{aligned}
\int\cos(\alpha x)\sin(\alpha x)\,dx &= \frac{-\cos^2(\alpha x)}{2\alpha}+c,\\
\int\cos^2(\alpha x)\,dx &= \frac{2\alpha x+\sin(2\alpha x)}{4\alpha}+c,\\
\int\sin^2(\alpha x)\,dx &= \frac{2\alpha x-\sin(2\alpha x)}{4\alpha}+c,\\
\int x\sin^2(\alpha x)\,dx &= \frac{x^2}{4}-\frac{x\sin(2\alpha x)}{4\alpha}-\frac{\cos(2\alpha x)}{8\alpha^2}+c,\\
\int x\cos^2(\alpha x)\,dx &= \frac{x^2}{4}+\frac{x\sin(2\alpha x)}{4\alpha}+\frac{\cos(2\alpha x)}{8\alpha^2}+c,\\
\int x^2\sin^2(\alpha x)\,dx &= \frac{x^3}{6}-\frac{x\cos(2\alpha x)}{4\alpha^2}x-\frac{\left(2\alpha^2x^2-1\right)\sin(2\alpha x)}{8\alpha^3}+c,\\
\int x^2\cos^2(\alpha x)\,dx &= \frac{x^3}{6}+\frac{x\cos(2\alpha x)}{4\alpha^2}+\frac{\left(2\alpha^2x^2-1\right)\sin(2\alpha x)}{8\alpha^3}+c
\end{aligned}$$

2D polar coordinates:

$$\begin{aligned}
x &= r\cos(\phi),\\
y &= r\sin(\phi),\\
r &= \sqrt{x^2+y^2},\\
\phi &= \arctan(y/x),\\
\mathbf{r} &= r\hat{\mathbf{r}},\\
\dot{\mathbf{r}} &= \dot{r}\hat{\mathbf{r}}+r\dot{\phi}\hat{\boldsymbol{\phi}},\\
\ddot{\mathbf{r}} &= (\ddot{r}-r\dot{\phi}^2)\hat{\mathbf{r}}+(r\ddot{\phi}+2\dot{r}\dot{\phi})\hat{\boldsymbol{\phi}}.
\end{aligned}$$

$$\begin{aligned}
i\hbar\frac{\partial}{\partial t}\psi(x,t) &= -\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2}\psi(x,t)+V(x)\psi(x,t),\\
E\psi(x) &= -\frac{\hbar^2}{2m}\frac{d^2}{dx^2}\psi(x)+V(x)\psi(x),\\
\Delta q &= \sqrt{\langle q^2\rangle-\langle q\rangle^2},\\
\Delta x\,\Delta p &\geq \tfrac{1}{2}|[x,p]| = \frac{\hbar}{2},\\
j_0(z) &= \frac{\sin z}{z},\\
n_0(z) &= -\frac{\cos z}{z},\\
\nabla^2 &= \frac{1}{r^2}\frac{\partial}{\partial r}r^2\frac{\partial}{\partial r}+\frac{1}{r^2\sin\theta}\frac{\partial}{\partial\theta}\sin\theta\frac{\partial}{\partial\theta}+\frac{1}{r^2\sin^2\theta}\frac{\partial^2}{\partial\phi^2}\\
-l(l+1)Y_l^m(\theta,\phi) &= \left(\frac{1}{\sin\theta}\frac{\partial}{\partial\theta}\sin\theta\frac{\partial}{\partial\theta}+\frac{1}{\sin^2\theta}\frac{\partial^2}{\partial\phi^2}\right)Y_l^m(\theta,\phi).
\end{aligned}$$

Pauli matrices:

$$\sigma_x=\begin{pmatrix}0&1\\1&0\end{pmatrix},\quad \sigma_y=i\begin{pmatrix}0&-1\\1&0\end{pmatrix},\quad \sigma_z=\begin{pmatrix}1&0\\0&-1\end{pmatrix},$$

$$[\sigma_j,\sigma_k]=2i\sum_l\epsilon_{jkl}\sigma_l,$$

$$\sigma_\pm=\tfrac{1}{2}(\sigma_x+i\sigma_y),$$

$$[\sigma_+,\sigma_-]=\sigma_z,$$

$$[\sigma_z,\sigma_\pm]=\pm 2\sigma_\pm,$$

Free evolution:

$$e^{i\omega a^\dagger at}ae^{-i\omega a^\dagger at}=e^{-i\omega t}a,$$

$$e^{i\Omega\sigma_z t/2}\sigma_\pm e^{-i\Omega\sigma_z t/2}=e^{\pm i\Omega t}\sigma_\pm.$$