3. (a) *Briefly* outline the use in scattering theory of

   (i) The Born Approximation;

   (ii) The method of partial waves;

   (iii) The Lippmann-Schwinger equation.

   Address the physical basis of the methods and their applicability with no more than a few sentences for each.

   (b) Describe the components of the wavefunction

$$\psi(\mathbf{x}) \sim e^{ikz} + f(\theta, \phi)\frac{e^{ikr}}{r}$$

   and the physical justification for interpreting it as the scattering solution of the time independent Schrödinger equation.

   (c) For a finite ranged *central* scattering potential, the scattering problem is considerably simplified. The wavefunction outside the potential can be expanded as

$$\begin{aligned}\psi(\mathbf{x}) &= \sum_l \psi_l(r) P_l(\cos\theta), \\ \psi_l(r) &= A_l \left[\cos\delta_l j_l(kr) - \sin\delta_l n_l(kr)\right],\end{aligned}$$

   where j_l and n_l are spherical Bessel and Neumann functions, and P_l are Legendre polynomials. Give a brief justification for this expansion.

   (d) Consider the *hard sphere potential*

$$V(r) = \begin{cases}\infty, & r < a, \\ 0, & r \geq a.\end{cases}$$

   (i) Use the above expansion and the boundary condition at $r = a$ to show that for any l, $\cot\delta_l = n_l(ka)/j_l(ka)$.

   (ii) The scattering length a_s and effective range r_0 are given by the low-k expansion

$$k\cot\delta_0 \approx -\frac{1}{a_s} + \frac{1}{2}r_0 k^2 + O(k^4).$$

   Using this expansion, find the scattering length and effective range for the hard sphere potential.